Calculus Warm Up #9- 5

1. Find c guarenteed by the Mean Value Theorem for

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f is cont & diff

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 - 4c = \frac{f(2) - f(-1)}{2+1}$$

$$3c^2 - 4c = \frac{-2 + 5}{3}$$

$$3c^2 - 4c = 1$$

$$3c^2 - 4c - 1 = 0$$

$$c = \frac{4 \pm \sqrt{16 - 4(3)(-1)}}{2(3)}$$

$$c = \frac{2 \pm \sqrt{7}}{3} \approx 1.5, -0.2$$

HW Questions: p. 392

25. $y = 4^x$

27. $y = 5^{x-2}$

29. $y = x^2 2^x$

31. $y = \log_3 x$

33. $y = \log_2\left(\dfrac{x^2}{x-1}\right)$

35. $y = \log_5 \sqrt{x^2 - 1}$

37. $y = \ln|x^2 - 1|$

$u = x^2 - 1$

$\frac{du}{dx} = 2x$

$y' = \frac{1}{x^2-1} \cdot 2x$

$y' = \frac{2x}{x^2-1}$

In Exercises 39–48, find dy/dx using logarithmic differentiation.

39. $y = x\sqrt{x^2 - 1}$

41. $y = \dfrac{x^2\sqrt{3x-2}}{(x-1)^2}$

$$\ln y = 2\ln x + \frac{1}{2}\ln(3x-2) - 2\ln(x-1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{3}{2(3x-2)} - \frac{2}{x-1}$$

combine
$LCD = 2x(x-1)(3x-2)$

$$= \frac{4(x-1)(3x-2) + 3x(x-1) - 4x(3x-2)}{2x(x-1)(3x-2)}$$

$$= \frac{4(3x^2-5x+2) + 3x^2 - 3x - 12x^2 + 8x}{2x(x-1)(3x-2)}$$

$$= \frac{3x^2 - 15x + 8}{2\cancel{x}(x-1)(3x-2)^1} \cdot \frac{x^{\cancel{2}}(3x-2)^{1/2}}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{x(3x^2-15x+8)}{2(x-1)^3\sqrt{3x-2}}$$

43. $y = \dfrac{x(x-1)^{3/2}}{\sqrt{x+1}}$

$$\ln y = \ln x + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}$$

combine $\to$ $LCD = 2x(x-1)(x+1)$
clean it up, then multiply
both sides by y.

45. $y = x^{2/x}$

$\ln y = \left(\frac{2}{x}\right)(\ln x)$

product rule!

47. $y = (x-2)^{x+1}$

$\ln y = (x+1)(\ln(x-2))$ product rule!

$y \cdot \frac{1}{y}\frac{dy}{dx} = \left[\frac{(x+1)}{x-2} + \ln(x-2)\right](x-2)^{x+1}$

another version of the answer:

$\frac{dy}{dx} = (x-2)^{x}\left[x+1+(x-2)\ln(x-2)\right]$

In Exercises 49 and 50, show that the given function is a solution to the differential equation.

Function	*Differential equation*
49. $y = 2\ln x + 3$	$x(y'') + y' = 0$

find y' & y''

then plug them into

In Exercises 51 and 52, find dy/dx by using implicit differentiation.

51. $x^2 - 3\ln y + y^2 = 10$

$2x - \frac{3}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(\frac{2y^2}{y} - \frac{3}{y}\right) = -2x$

$\frac{dy}{dx} = \frac{-2x}{1} \cdot \frac{y}{2y^2-3}$

$\frac{dy}{dx} = \frac{-2xy}{2y^2-3}$

In Exercises 53 and 54, find an equation of the tangent line to the graph of the equation at the given point.

Equation	*Point*
53. $y = 3x^2 - \ln x$	$(1, 3)$

In Exercises 55–60, find any relative extrema and inflection points, and sketch the graph of the function.

55. $y = \dfrac{x^2}{2} - \ln x$

57. $y = x(\ln x)$

Domain: $x > 0$

$y' = x \cdot \frac{1}{x} + (1)\ln x$

$0 = 1 + \ln x$

$-1 = \ln x$

$x = e^{-1}$

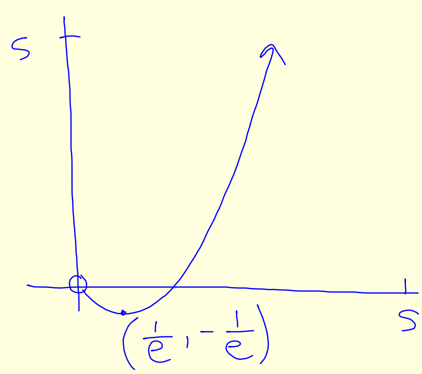
f': 0.1, 1

0 ⊖ $\frac{1}{e}$ ⊕

Min @ $\left(\frac{1}{e}, -\frac{1}{e}\right)$

$y'' = \frac{1}{x}$

always positive so always concave up.



59. $y = \dfrac{x}{\ln x}$

HW: finish p. 392 homework and AP Review #5

AP Review #5 due
turned in on Tuesday

Some answers posted after this slide

Answers #5:

4a) Show clear process

b) show derivative $= 0$ when $x = 3$

y-coord of $P = 2$

c) $\frac{d^2y}{dx^2} = \left[(8y-3x)(3\frac{dy}{dx}-2)-(3y-2x)(8\frac{dy}{dx}-3)\right] / (8y-3x)$

plug in $x=3$, $y=2$, $\frac{dy}{dx}=0$

$\frac{d^2y}{dx^2}$ @ $(3,2)$ $= -\frac{8}{7}$

concave down confirms Max @ $(3,2)$

5a) Show clear process

b) $\frac{1}{2} = \frac{y}{2y-x}$

$\downarrow$

$x = 0$

points: $(0, \pm\sqrt{2})$

c) Set $\frac{dy}{dx} = 0$

$\downarrow$

$y = 0$

but $y \neq 0$ in original (show this)

d) Find the related rates equation.

Find x when $y = 3$ in the original.

Plug in everything you know...

$\frac{dx}{dt} = \frac{22}{3}$